WEAKLY RELATIVISTIC DIELECTRIC TENSOR FOR ARBITRARY WAVENUMBERS

F. Volpe and N.B. Marushchenko

Max-Plank-Institut für Plasmaphysik, EURATOM-Association,
Greifswald, Germany

e-mail: frv@ipp.mpg.de

The relativistic dielectric tensor was obtained by Trubnikov as a steady-state solution of the linearised Vlasov-Maxwell problem for a relativistic plasma [1]. A more treatable weakly relativistic approximation was then derived by Dnestrovskii for perpendicular propagation and extended by Shkarofsky to quasi-perpendicular incidence [2], such that $N_{||} \ll \beta_T \ll 1$ and $|N_{||}(2\delta - N_{||}^2)| \ll \beta_T^2$. Here β_T is the electron thermal velocity in c units and $\delta = (\omega - n\omega_c)/\omega$ indicates the distance from the n-th cyclotron harmonic in dimensionless units. Assumptions of small wavenumbers parallel to the ambient magnetic field, $N_{||}$, are generally violated by electron Bernstein waves generated via ordinary-extraordinay-Bernstein (O-X-B) mode conversion of an *obliquely* injected O-mode [3, 4] or detected by *oblique* observation of the reverse, B-X-O, process [5, 6]. Besides, as a consequence of the large $N_\perp$ and of the evolution of the ray in a bent magnetic field, $N_{||}$ can grow very fast and reach large values of order 1, even starting from small values at the antenna [4, 7]. Finite $N_{||}$ are also important for ion Bernstein waves, and were previously treated numerically with the aid of root finders. Finally the (mild) relativistic mass gain is obviously important for electromagnetic electron cyclotron (EC) waves, as it resolves the EC emission/absorption line. In the $N_{||} \ll \beta_T$ limit, this mechanism dominates over Doppler broadening and is satisfactorily described by Shkarofsky functions. In the opposite limit, relativistic effects are neglected and the warm non-relativistic dielectric tensor can be utilized. This includes Doppler broadening only. The fully relativistic tensor ϵ embodies both the broadening mechanisms, thus in principle it suits intermediate angles such that $N_{||} \approx \beta_T$, however it is very complicated, not in a closed form and can only be evaluated numerically. Motivated by these considerations, a semi-relativistic approximation of Trubnikov's fully relativistic tensor is derived in the present work by means of a simple Taylor-expansion of the Lorentz factor and without making any specific assumption on $N_{||}$ and $N_\perp$. The results extend the validity of Shkarofsky's treatment, previously restricted to quasi-perpendicular incidence, and permit to handle cases in which Doppler and relativistic widths of electron cyclotron resonances are comparable. Also, as $N_\perp$ is arbitrary, the proposed approximation might find application in numerical simulations of Bernstein waves, characterised by $N_\perp \approx 1/\beta_T \geq 1$. Finally, for ease of calculation and to emphasize the link with previous results, it is shown that the new formulas can be put in the form of Shkarofsky functions with shifted argument and modified width.

References

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